Multiple Phase Flow

Burhan S Abdulrazak, Ph.D.
Chemical Engineering Department

Lecture Notes: Lagrangian Tracking of a Single Particle Under Different Forces

Introduction

Lagrangian particle tracking involves following the trajectory of an individual particle through a fluid flow field by solving the equations of motion. This approach provides detailed insights into the dynamics of a particle as it interacts with the surrounding flow and experiences various forces.

Applications of Lagrangian particle tracking include:

- Sedimentation studies
- Aerosol dynamics
- Multiphase reactor analysis
- Environmental modeling (e.g., pollutant dispersion)

Key Concepts

1. Lagrangian Framework

The particle's position and velocity are tracked over time as it moves through a flow field. Its trajectory is governed by Newton's second law:

$$m_p \frac{du_p}{dt} = \sum F$$

Where:

- m_p: Particle mass
- u_p: Particle velocity
- $\sum F$: Sum of forces acting on the particle

2. Assumptions

- The particle is treated as a discrete entity.
- The fluid flow field is known (e.g., from computational fluid dynamics or experimental data).
- Forces acting on the particle are computed based on the local flow field and particle properties.

Forces Acting on a Particle

Drag Force (F_D):

- Resistive force due to relative motion between the particle and fluid.
- Given by:

$$F_D = \frac{1}{2} C_D \rho_f A_p \ | u_r | ur$$

Where:

- C_D: Drag coefficient
- ρ_f: Fluid density
- A_p: Projected area of the particle
- $u_r = u_p - u_f$: Relative velocity between particle (u_p) and fluid (u_f)

Gravitational Force (F_g):

- Acts on the particle due to its weight.
- Given by:

$$F_g = m_p g$$

- Where g is the acceleration due to gravity.

Buoyancy Force (F_b):

- Upward force due to displaced fluid.
- Given by:

$$F_b = -\rho_f V_p g$$

- Where V_p is the particle volume.

Virtual Mass Force (F_{vm}):

- Represents the added inertia of the fluid as the particle accelerates.
- Given by:

$$F_{vm} = C_{vm}\,\rho_f\,V_p\,\frac{du_f}{dt}$$

- Where C_{vm} is the virtual mass coefficient.

Basset History Force (F_h):

- Accounts for the history of viscous effects as the particle accelerates.
- Given by:

$$F_h = 6\,\pi\,\mu\,R_p \int_0^t \frac{du_p}{d\tau}\,\frac{1}{\sqrt{t-\tau}}\,d\tau$$

- Where μ is the fluid viscosity and R_p is the particle radius.

Lift Force (F_L):

- Acts perpendicular to the direction of motion due to shear or rotation in the flow.
- Given by:

$$F_L = C_L \, \rho_f \, A_p \, | ur |^2$$

Where C_L is the lift coefficient.

Equations of Motion

The particle's motion is described by:

Translational Motion:

$$m_p \frac{du_p}{dt} = F_g + F_b + F_D + F_{vm} + F_h + F_L$$

Rotational Motion (if applicable):

$$I_p \frac{d\omega_p}{dt} = \mathrm{M}$$

Where:

- I_p: Particle moment of inertia
- ω_p: Angular velocity
- M: Torque acting on the particle

Numerical Solution

Integration of Equations:

- Use numerical methods (e.g., Runge-Kutta) to solve the equations of motion.
- Track the particle's position and velocity over time.

Time Step Consideration:

- Time step must resolve the smallest time scale in the problem (e.g., fluid-particle interaction timescale).

Coupling with Fluid Flow:

- Update fluid velocity (u_f) at the particle's location if the flow is unsteady.

Applications

Sedimentation and Settling:

- Predicts particle settling velocity in liquids.
- Useful in wastewater treatment and mineral processing.

Aerosol Dynamics:

- Tracks particle trajectories in airflows for environmental and industrial processes.

Spray and Droplet Analysis:

- Models droplet breakup, evaporation, and transport in sprays.

Fluidized Beds:

- Simulates particle movement and mixing in gas-solid reactors.

Pollutant Dispersion:

- Tracks transport of particles in environmental modeling.

Example Problem

Problem Statement:

A spherical particle of diameter D=0.01 m, density ρ_p=2500 kg/m^3, is released in air (ρ_f = 1.2 kg/m^3, μ=1.8×10^{-5} Pa.s at an initial velocity u_p= 0 m. Gravity acts downward (g=9.81 m/s^2). Calculate the terminal velocity (u_t) of the particle.

Solution:

At terminal velocity, drag force equals the net gravitational force:

$$F_D = F_g - F_b$$

Gravitational Force:

$$F_g = m_p g = \frac{\pi D^3}{6} \rho_p g$$

$$F_g = \frac{\pi (0.01)^3}{6} \cdot 2500 \cdot 9.81 \approx 0.0322 \ N$$

Buoyancy Force:

$$F_b = \frac{\pi D^3}{6}\rho_f g$$

$$F_b = \frac{\pi(0.01)^3}{6} \cdot 1.2 \cdot 9.81 \approx 0.000015\, N$$

Net Force:

$$F_{net} = F_g - F_b = 0.0322 - 0.000015 \approx 0.0322\, N$$

Drag Force at Terminal Velocity:

$$F_D = \frac{1}{2} C_D\, \rho_f A\, u_t^2$$

Solve for u_t using $\mathrm{C_D} =\ 0.44$ (valid for high Re):

Substituting:

$$A = \frac{\pi D^2}{4} = \frac{\pi(0.01)^2}{4} \approx 7.85 \times 10^{-5}\, m^2$$

$$u_t = \frac{2 \cdot 0.0322}{0.44 \cdot 1.2 \cdot 7.85 \times 10^{-5}} \approx 10.2\ \frac{m}{s}$$

Conclusion

Lagrangian tracking provides detailed insights into the behavior of individual particles under various forces. By incorporating drag, buoyancy, gravity, and other forces, this method is crucial for predicting particle trajectories and dynamics in complex systems.